Stability Analysis of Discrete Hopfield Neural Networks With Delay and Its Application

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Abstract- *Discrete Hopfield Neural Networks (DHNNs) with delay, which can deal with temporal information, are a generalization of the DHNNs without delay. This paper investigates the convergence theorems in DHNNs with delay. We present two generalized updating rules, one for serial mode and the other for parallel mode. The convergence speed of these proposed updating rules is faster than existing updating rules. By means of the new network structure and its convergence theorems, we propose a local searching algorithm for combinatorial optimization. We also relate the maximum value of a bivariate energy function to the stable states of the DHNNs with delay. Furthermore, we describe an algorithm for the DHNNs with delay in which the delay term is regarded as noise, which has a higher convergence rate than usual algorithms in the Hopfield neural network without delay. One application is presented to demonstrate the higher rate of convergence of our algorithm.*

Keywords: Discrete Hopfield neural network, Delay, Convergence, Stable state.

1 Introduction

Recently, the convergence of various connectionist models has been investigated by researchers in many fields. Since Hopfield and Tank first applied Continuous Hopfield Neural Networks CHNNs to the traveling salesman problem[1], CHNNs have been demonstrated to have the capability providing powerful approaches for a wide variety of combinatorial optimization problems [2]. However, Hopfield and Tank's original approach has been subjected to severe criticism, most notably from [3]. To overcome the deficiencies of the Hopfield network, various attempts have been made to optimally select the parameters of the network and eliminate infeasible solutions. Among these attempts, some approaches maintain the original structure of the Hopfield network while others involve changing the structure. Four methods exist to correct the deficiencies of DHNNs and CHNNs. The first method involves changing the updating rule to a generalized updating rule [4]. The second method involves adding noise to the network [5]. The third method involves adjusting the parameters of the network [6]. The fourth method involves changing the structure of the network and adding corresponding disturbance algorithms [7]. Specifically, Takefuji et al. [8] found that DHNNs are computationally more efficient in comparison with CHNNs. CHNNs and DHNNs have been proposed as models for solving a wide variety of problems in fields as diverse as associative memory devices, pattern recognition, the extraction of knowledge, and combinatorial optimization [2,9,10]. The properties of convergence are the foundation of real applications, such as in the extraction of knowledge and combinatorial optimization problems.

There are several motivations for investigating the convergence of DHNNs. These include understanding how DHNNs handle (Fuzzy) feature selection and extraction, quantifying how the features have optimal saliencies, and designing a new neural network model to perform the task of optimizing a fuzzy evaluation index [11]. In this paper, we mainly investigate DHNNs with delay. We provide analyses for DHNNs with delay, for both serial and parallel updating modes. Based on the reduction of the energy function and by means of a modified DHNN as well as its updating mode, we obtain the convergence conditions of DHNNs with delay and the updating steps. In addition, we discuss the application of the convergence of DHNNs with delay to a min-cut problem.

This paper has the following organization. In this section (Section 1), we provide the introduction. In Section 2, we give a brief review of the DHNNs without and with delay. In Section 3, we introduce some definitions and the notation used in this paper. In Section 4, we investigate the convergence of the DHNNs with delay and we present our main convergence results. The differences between a DHNN with delay and without delay are given in Section 5. In Section 6, we discuss one real-world application of the convergence of the DHNNs with delay: the minimum-cut (MC) problem. The last section offers the conclusions of this paper.

2 Hopfield Networks

DHNN without delay can be viewed as a graph in which each node represents a neuron in the network. The order of the network is equal to the number of neurons in the network. The strength of the connection between each

d) $w_{ii} \ge \sum_{\substack{j=1 \\ j\neq i}}^{n} \left|\alpha_i w_{ij} - w_{ji}\right| = \sum_{\substack{j=1 \\ j\neq i}}^{n} \left(1-\alpha_j\right)\left|w_{ji}\right|$

$$= \sum_{\substack{j=1 \\ j\neq i}}^{n} \beta_j \left|w_{ji}\right|, i = 1, 2, \ldots n.$$

After conditions a) to d) are satisfied (i.e., a new neural network structure is constructed), we need to operate the neural network in a serial mode, for every initial state, to attain a stable state. Each stable state is one of the local maxima (or it may be the global maximum) of $Max\ \{Q_c(X)\}$. The following experimental results are based on randomly choosing a graph (the corresponding order of the graph is from 6 to 32) 100 times, which is then used as a DHNN without delay and as a DHNN with delay. In Fig. 1, it can be seen that our DHNN with delay converges faster than a DHNN without delay as they seek a solution to $Max\ \{Q_c(X)\}$. However, the rate to reach submax is slower than Hopfield's.

Randomly chosen graph

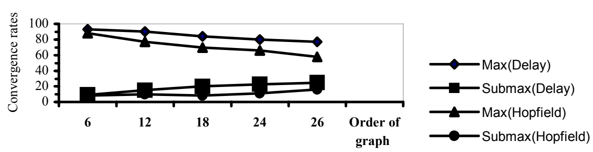

Figure 1.Exprimental results

7 Conclusions

In this paper we have established a new energy function that is associated with a DHNN with delay, and we have proposed a new evolutional mode. Sufficient conditions are given for the convergence of a DHNN with delay, which generalizes earlier results. The theorem for convergence provides guidelines for constructing a feasible rule evaluation function. The improved performance is given for the time required for convergence. The algorithm using a DHNN with delay is given in different forms. We apply this algorithm to the problem of finding a minimum cut (MC).

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